Summer Assignment

Name: ______________________ Due Date: ________

◆ Skill A Writing an equation of a line in slope-intercept form

Recall The slope-intercept form of a line is $y = mx + b$.

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$$

◆ **Example**

Write an equation for each line.

a. containing (0, 1) and with a slope of -2

b. containing (3, -4) and (9, 0)

◆ **Solution**

a. The slope, m, is given as -2. The line contains (0, 1), so this point is the y-intercept, or b is 1. Substituting these numbers into the equation gives $y = -2x + 1$.

b. First find the slope. $m = \frac{-4 - 0}{3 - 9} = \frac{-4}{-6} = \frac{2}{3}$

Then subtitute the coordinates of one of the given points into the equation and solve for b.

For the point (9, 0): $0 = \frac{2}{3}(9) + b$

$0 = 6 + b$

$b = -6$

Substituting this number for b and $\frac{2}{3}$ for m into the equation $y = mx + b$ gives the equation $y = \frac{2}{3}x - 6$.

For each equation, find the slope and the y-intercept.

1. $y = 3x - 1$ ________ **2.** $y = \frac{1}{2}x + 2$ ________ **3.** $y = -x + \frac{1}{2}$ ________

Write an equation in slope-intercept form for each line.

4. with a slope of 2 and a y-intercept of -1 ________

5. containing (0, -3) and with a slope of $\frac{1}{3}$ ________

Write an equation in slope-intercept form for the line that contains each pair of points.

6. (1, 1) and (3, 5) ________ **7.** (2, -4) and (-1, 5) ________

8. (2, 4) and (-4, 1) ________ **9.** (1, 0) and (3, 2) ________

◆ Skill B Writing an equation of a line in point-slope form

Recall The point-slope form for an equation of a line is $y - y_1 = m(x - x_1)$.

◆ **Example**

Write an equation for the line through (1, -1) and (-1, 5)

a. in point-slope form.

b. in slope-intercept form.

◆ **Solution**

a. First find m.

$$m = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{-1 - 5}{1 - (-1)} = \frac{-6}{2} = -3$$

Substitute the slope and one of the points into the point-slope equation.

$y - y_1 = m(x - x_1)$

$y - (-1) = -3(x - 1)$ Use the point (1, -1).

$y + 1 = -3(x - 1)$ Simplify.

b. Rewrite the equation in the form $y = mx + b$.

$y + 1 = -3(x - 1)$

$y + 1 = -3x + 3$ Distributive Property

$y = -3x + 2$ Subtract 1 from each side.

Write an equation for each line in point-slope form.

1. containing (4, -1) and with a slope of $\frac{1}{2}$ ________

2. crossing the x-axis at $x = -3$ and the y-axis at $y = 6$ ________

3. containing the points (-6, -1) and (3, 2) ________

Rewrite each equation in slope-intercept form.

4. the line from Exercise **1.** ________

5. the line from Exercise **2.** ________

6. the line from Exercise **3.** ________

7. In what situations would you find it easier to use point-slope form, and in what situations would you find it easier to use slope-intercept form? ________

◆ Skill D Find the zeros of a polynomial function by factoring

Recall The zeros of a function are the values of x that make y equal to 0.

◆ **Example 1**

Find the zeros of the function $y = (x - 2)(x + 5)$.

◆ **Solution**

Let $y = 0$. Then use the Zero-Product Property to solve for x.

$$(x-2)(x+5) = 0$$
$$(x-2) = 0 \quad \text{or} \quad (x+5) = 0$$
$$x = 2 \quad \text{or} \quad x = -5$$

The zeros of $y = (x - 2)(x + 5)$ are 2 and -5.

Recall A quadratic polynomial can be factored into two binomials.

◆ **Example 2**

Solve the equation $x^2 - x - 6 = 0$.

◆ **Solution**

Since $x^2 - x - 6$ can be factored into $(x + 2)(x - 3)$, you can rewrite $x^2 - x - 6 = 0$ as $(x + 2)(x - 3) = 0$. Solve the equation $(x + 2)(x - 3) = 0$.

$$x + 2 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -2 \quad \text{or} \quad x = 3$$

The solutions to $x^2 - x - 6 = 0$ are -2 and 3.

Solve by factoring.

1. $x^2 - 4x - 12 = 0$ __________
2. $x^2 - 6x + 9 = 0$ __________
3. $x^2 - 9x + 14 = 0$ __________
4. $x^2 + 6x + 5 = 0$ __________
5. $x^2 - 7x + 10 = 0$ __________
6. $x^2 - 36 = 0$ __________
7. $x^2 + 8x + 16 = 0$ __________
8. $x^2 - x - 12 = 0$ __________
9. $9x^2 - 1 = 0$ __________
10. $4x^2 + 4x + 1 = 0$ __________

◆ Skill E Using the quadratic formula to solve equations

Recall The solutions for a quadratic equation written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be found by using the quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

◆ **Example**

Use the quadratic formula to solve $x^2 - 8x + 15 = 0$ for x.

◆ **Solution**

For $x^2 - 8x + 15 = 0$, a is 1; b is -8, and c is 15. Substitute these values in the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4)(1)(15)}}{(2)(1)}$$
$$= \frac{8 \pm \sqrt{64 - 60}}{2}$$
$$= \frac{8 \pm \sqrt{4}}{2}$$
$$= \frac{8 \pm 2}{2}$$
$$x = 3 \text{ or } x = 5$$

The solutions are 3 and 5.

Use the quadratic formula to solve each equation.

1. $x^2 - 5x + 4 = 0$ __________
2. $x^2 - 2x - 24 = 0$ __________
3. $x^2 + 6x + 9 = 0$ __________
4. $x^2 + 3x - 10 = 0$ __________
5. $2x^2 - x - 6 = 0$ __________
6. $2x^2 + x - 4 = 0$ __________

◆ Skill H Writing and evaluating functions

Recall The value of $f(x) = x^2 + 5$ depends on the value of x.

◆ **Example 1**

Sarah uses an internet server which charges \$12.50 per month plus \$0.60 for each hour over 20 hours that she uses it during the month. Write this relation in function notation. How much will she be charged for using the service for 38 hours in April?

◆ **Solution**

Let h = number of hours over 20. Thus, the function is as follows.

$f(h) = 12.50 + 0.60h$

$f(18) = 12.50 + 0.60(18)$ *where h = 18*

$f(18) = 23.30$

The charge for April will be \$35.30.

◆ **Example 2**

If $g(x) = x^2 + 3x$, find $g(-5)$.

◆ **Solution**

$g(-5)$ means replace x with the value -5 and evaluate $g(x)$.

$$\begin{aligned} g(-5) &= (-5)^2 + 3(-5) \\ &= 25 - 15 \\ &= 10 \end{aligned}$$

Thus, $g(-5) = 10$.

Let $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$. Evaluate each function.

1. $f(6)$ ____________ **2.** $f(0)$ ____________

3. $f\left(\frac{1}{2}\right)$ ____________ **4.** $g(1)$ ____________

5. $g(-2)$ ____________ **6.** $g\left(\frac{1}{2}\right)$ ____________

7. $f(1) + g(0)$ ____________ **8.** $g(4) - f(5)$ ____________

9. $f(0) \cdot g(0)$ ____________ **10.** $g(-6) \cdot f(-6)$ ____________

◆ Skill J Finding the composite of two functions

Recall To write an expression for the composite function $(f \circ g)(x)$, replace each x in the expression for f with the expression defining g. Then simplify the result.

◆ **Example**

Let $f(x) = 5x$ and $g(x) = 2x^2 - 3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$. Then write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

◆ **Solution**

$(f \circ g)(2)$: $g(2) = 2(2)^2 - 3 = 5$ $f(g(2)) = f(5) = 5(5) = 25$

Thus, $(f \circ g)(2) = 25$.

$(g \circ f)(2)$: $f(2) = 5(2) = 10$ $g(f(2)) = g(10) = 2(10)^2 - 3 = 197$

Thus, $(g \circ f)(2) = 197$.

To write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$, use the variable x instead of a particular number.

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(2x^2 - 3) \\ &= 5(2x^2 - 3) \\ &= 10x^2 - 15 \end{aligned} \qquad \begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(5x) \\ &= 2(5x)^2 - 3 \\ &= 50x^2 - 3 \end{aligned}$$

Let $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$. Find each composite function.

1. $(f \circ g)(x)$ ____________ **2.** $(g \circ f)(x)$ ____________

3. $(h \circ f)(x)$ ____________ **4.** $(h \circ g)(x)$ ____________

5. $(g \circ g)(x)$ ____________ **6.** $(h \circ h)(x)$ ____________

7. $(g \circ h)(4)$ ____________ **8.** $(f \circ f)(-3)$ ____________

9. $(f \circ (g \circ h))(1)$ ____________ **10.** $(g \circ (g \circ g))(5)$ ____________

◆ Skill L Graphing piecewise, step, and absolute-value functions

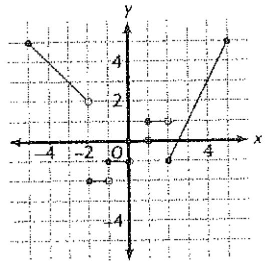
Recall A piecewise function in x is a function defined by different expressions in x on different intervals for x.

◆ **Example**

Graph this piecewise function.

$$f(x) = \begin{cases} |x|, & \text{if } -5 \le x < -2 \\ [x], & \text{if } -2 \le x < 2 \\ 2x - 5, & \text{if } 2 \le x \le 5 \end{cases}$$

◆ **Solution**



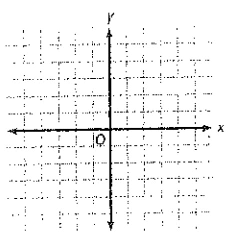
x	−5	−4	−3	−2.5
$y = \lvert x \rvert$	5	4	3	2.5

x	−2	−1.5	−1	−0.5	0	1
$y = [x]$	−2	−2	−1	−1	0	1

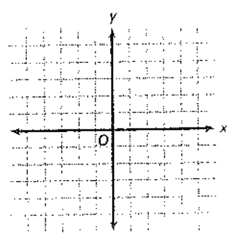
x	2	2.5	3	4	5
$y = 2x - 5$	2	0	1	3	5

Graph each function.

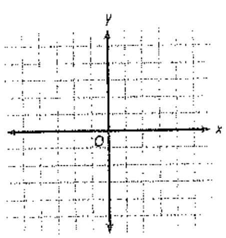
1. $f(x) = \begin{cases} x + 3, \text{ if } x < 0 \\ -2x + 5 \text{ if } x \ge 0 \end{cases}$



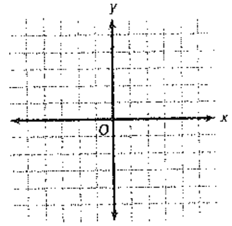
2. $f(x) = \begin{cases} \frac{1}{2}x \text{ if } -4 \le x \le 2 \\ 2x - 3 \text{ if } x > 2 \end{cases}$



3. $f(x) = \begin{cases} |x| \text{ if } x \le 1 \\ 2 - |x - 2| \text{ if } x > 1 \end{cases}$



4. $f(x) = \begin{cases} [x] \text{ if } -2 \le x \le 1 \\ \lceil x \rceil \text{ if } 1 < x \le 4 \end{cases}$



◆ Skill M Using logarithms to solve exponential equations

Recall The common logarithm, $\log_{10} x$, is usually written as $\log x$.

◆ **Example**

Solve each equation.

a. $3^x = 81$ **b.** $5^x = 75$ **c.** $7^{x+1} = 150$

◆ **Solution**

a. $3^x = 81$

Since 81 is a power of 3, use powers of 3.

$3^x = 3^4$

$x = 4$ *One-to-One Property of Exponential Functions*

b. $5^x = 75$

Since 75 is **not** a power of 5, use logarithms to solve this equation.

$\log 5^x = \log 75$

$x \log 5 = \log 75$ *Power Property of Logarithms*

$x = \dfrac{\log 75}{\log 5}$

$x \approx 2.68$

Check: $5^{2.68} \approx 75$

c. $7^{x+1} = 150$

$\log 7^{x+1} = \log 150$

$(x + 1)\log 7 = \log 150$

$x + 1 = \dfrac{\log 150}{\log 7}$

$x = \dfrac{\log 150}{\log 7} - 1$

$x \approx 1.57$

Solve each equation. Round your answers to the nearest hundredth.

1. $7^x = 80$ __________
2. $5^x = 10$ __________
3. $6^x = 1296$ __________
4. $4^{x+1} = 100$ __________
5. $2^{x-3} = 25$ __________
6. $3^{x+4} = 27$ __________
7. $6^{2x-7} = 216$ __________
8. $5^{3x-1} = 49$ __________
9. $10^{x+5} = 125$ __________

◆ **Skill O** Using the inverse functions $f(x) = e^x$ and $g(x) = \ln x$ to solve equations

Recall $\ln e$ is the base e logarithm of x. Therefore, $\ln e = 1$, just like $\log 10 = 1$ (base 10).

◆ **Example 1**

Simplify each expression.

a. $e^{\ln 4}$ **b.** $\ln e^2$

◆ **Solution**

a. Since $y = e^x$ and $y = \ln x$ are inverse functions, $e^{\ln x} = x$. So, $e^{\ln 4} = 4$.

b. Because of inverse functions, $\ln e^x = x$. So $\ln e^2 = 2$.

◆ **Example 2**

Solve for x.

a. $2e^{2x+1} = 60$ **b.** $\ln x = 3.2$

◆ **Solution**

a.
$$\begin{aligned} 2e^{2x+1} &= 60 \\ e^{2x+1} &= 30 \\ \ln e^{2x+1} &= \ln 30 \\ 2x + 1 &= \ln 30 \\ x &= \frac{\ln 30 - 1}{2} \\ x &\approx 1.20 \end{aligned}$$

b.
$$\begin{aligned} \ln x &= 3.2 \\ e^{\ln x} &= e^{3.2} \\ x &= e^{3.2} \\ x &\approx 24.53 \end{aligned}$$

Simplify each expression.

1. $e^{\ln 4}$ ________
2. $e^{\ln 15}$ ________
3. $e^{2 \ln 3}$ ________
4. $\ln e^9$ ________
5. $\ln e^5$ ________
6. $5 \ln e^3$ ________

Solve each equation for x by using the natural logarithmic function.

7. $e^x = 34$ ________
8. $3e^x = 120$ ________
9. $e^x - 8 = 51$ ________
10. $\ln x = 2.5$ ________
11. $\ln(3x - 2) = 2.8$ ________
12. $\ln e^x = 5$ ________

◆ **Skill Q** Understanding the effect of order on combining transformations

Recall To determine the order of transformations to a function, reverse the order of operations. Addition or subtraction indicates a vertical translation; multiplication or division indicates a vertical stretch; addition or subtraction within parentheses or within absolute-value symbols indicates a horizontal translation.

◆ **Example**

Describe the various transformations included in the equation $y = 2|x - 1| + 3$.

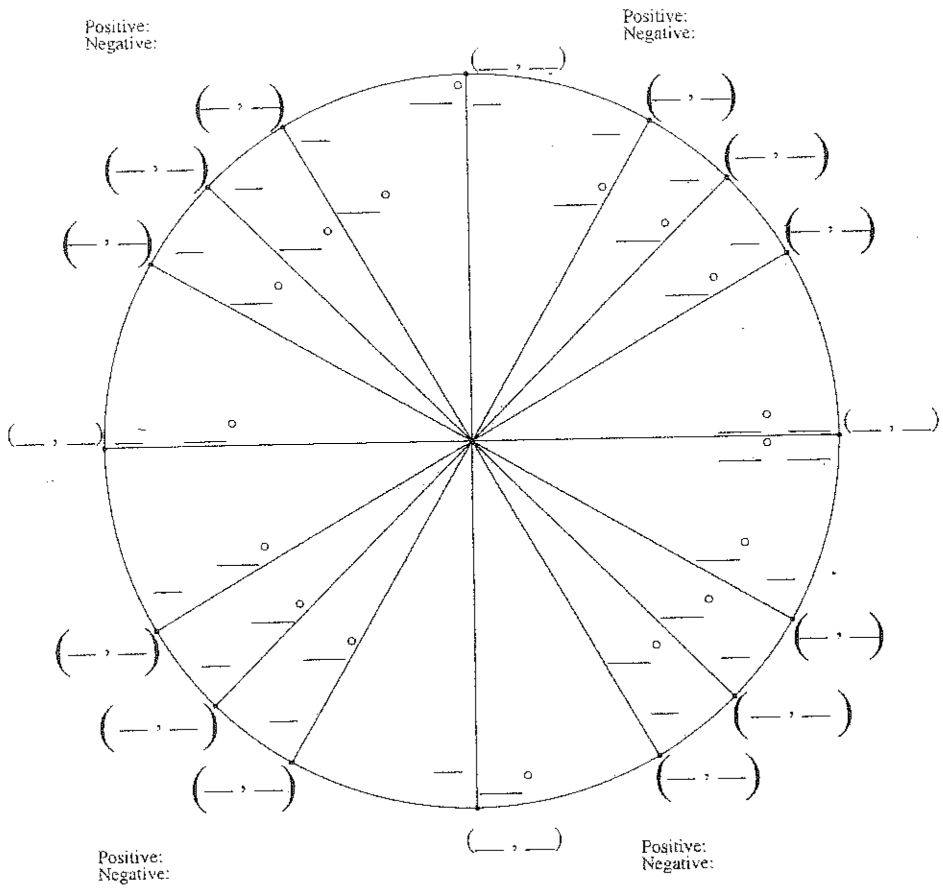
◆ **Solution**

The first operation to consider is the addition of 3. This affects the parent function by translating it vertically 3 units up. The second operation, multiplication by 2, stretches the translated function by a factor of 2. The third operation, subtraction of 1, translates the stretched function horizontally 1 unit to the right. Thus, the parent function, $y = |x|$, has been shifted 1 unit to the right, stretched by a factor of 2, and then shifted 3 units up.

Describe the transformations of the parent functions included in each equation.

1. $y = -3|x + 2| - 3$ ________
2. $y = 2(x - 3)^2 + 1$ ________
3. $y = 4|x - 1| + 2$ ________
4. $y = 4 \cdot 2^x - 2$ ________

Fill in The Unit Circle



Evaluate each expression. Give exact answers.

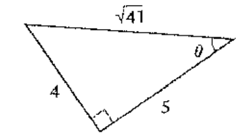
1. $\sin \frac{3\pi}{4}$ ______
2. $\cos \frac{2\pi}{3}$ ______
3. $\tan \frac{5\pi}{6}$ ______
4. $\cos\left(-\frac{7\pi}{6}\right)$ ______
5. $\tan\left(-\frac{\pi}{4}\right)$ ______
6. $\sin \pi$ ______

◆ Skill Y Finding the trigonometric functions of an acute angle

Recall The hypotenuse is the longest side in a right triangle and is opposite the right angle.

◆ **Example**

Refer to the triangle shown at right and give values for $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.



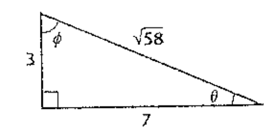
◆ **Solution**

The hypotenuse (hyp.) has a length of $\sqrt{41}$.
The leg opposite (opp.) θ has a length of 4.
The leg adjacent (adj.) to θ has a length of 5.

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{4}{\sqrt{41}}$ $\quad \csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{41}}{4}$ $\quad \cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{\sqrt{41}}$

$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{41}}{5}$ $\quad \tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{5}$ $\quad \cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{5}{4}$

Refer to the triangle at right to find each value. Give exact answers.



1. $\sin\theta$ ______
2. $\cos\theta$ ______
3. $\tan\theta$ ______
4. $\csc\theta$ ______
5. $\sec\theta$ ______
6. $\cot\theta$ ______
7. $\sin\phi$ ______
8. $\cos\phi$ ______
9. $\tan\phi$ ______
10. $\csc\phi$ ______

Convert the following degree measures to radian measures. Give exact answers.

1. 270° ______
2. 45° ______
3. 225° ______
4. 210° ______
5. −90° ______
6. −300° ______

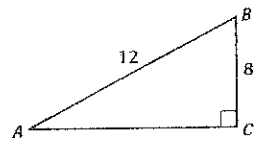
Convert each of the following radian measures to degree measures.

7. $\frac{\pi}{4}$ ______
8. $\frac{3\pi}{2}$ ______
9. $\frac{5\pi}{6}$ ______
10. $\frac{5\pi}{3}$ ______
11. -3π ______
12. $-\frac{11\pi}{6}$ ______

◆Skill AA Solving a right triangle

Recall *Solving a triangle* means to use given measures to find the unknown measures of the other sides and angles of the triangle.

◆ **Example 1**
Solve the triangle shown at right.



◆ **Solution**
Using the hypotenuse and side opposite $\angle A$,

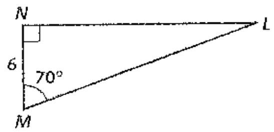
$\sin \angle A = \frac{8}{12}$

$m\angle A = \sin^{-1}\left(\frac{8}{12}\right) \approx 42°$ *Round to the nearest whole degree.*

$m\angle A + m\angle B = 90°$
$42° + m\angle B = 90°$
$m\angle B \approx 48°$

$8^2 + (AC)^2 = 12^2$
$AC = \sqrt{12^2 - 8^2}$
$AC \approx 8.9$

◆ **Example 2**
Solve the triangle shown at right.



◆ **Solution**
Using the side adjacent to $\angle M$,

$\cos 70° = \frac{6}{LM}$, where LM is the hypotenuse.

$LM = \frac{6}{\cos 70°} \approx 17.5$

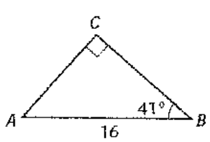
$(LN)^2 + (MN)^2 = (LM)^2$

$LN \approx \sqrt{17.5^2 - 6^2} \approx 16.4$

$m\angle L + m\angle M = 90°$
$m\angle L + 70° = 90°$
$m\angle L = 20°$

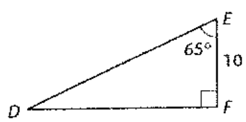
Solve each triangle. Round each angle measure to the nearest degree and each side length to the nearest tenth.

1.



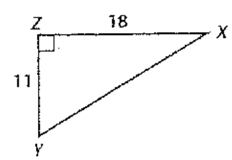
____

2.



____

3.

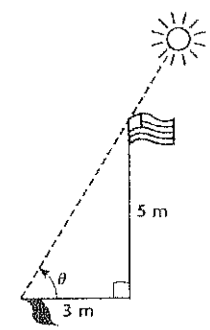


____

◆Skill CC Applying inverse trigonometric functions

Recall $\sin \theta = \frac{\text{opp.}}{\text{hyp.}}$ $\cos \theta = \frac{\text{adj.}}{\text{hyp.}}$ $\tan \theta = \frac{\text{opp.}}{\text{adj.}}$

◆ **Example**
At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long. What is the angle of elevation of the sun at this time?



◆ **Solution**
Since 3 meters is the length of the side **adjacent** to θ and 5 meters is the length of the side **opposite** θ, use the tangent function.

$\tan \theta = \frac{5}{3}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right)$

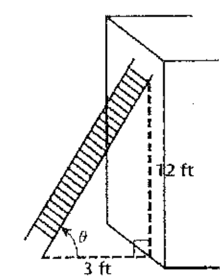
This last equation states that θ is the angle that has a tangent of $\frac{5}{3}$.

$\theta \approx 59°$ *Use calculator in **degree** mode.*

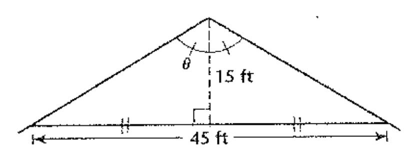
Find the measure of each angle to the nearest whole degree.

1. Find the measure of the smallest angle in a right triangle with sides of 3, 4, and 5 centimeters. ______________

2. What is the angle between the bottom of the ladder and the ground as shown at right? ______________



3. Find the angle at the peak of the roof as shown at right. ______________



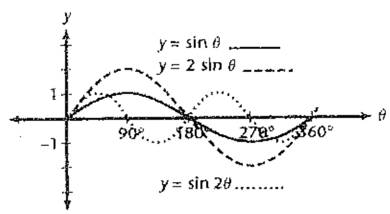
4. The hypotenuse of a right triangle is 3 times as long as the shorter leg. Find the measure of the angle between the shorter leg and the hypotenuse. ______________

◆Skill DD Graphing functions of the form $y = a \sin b\theta$, $y = a \cos b\theta$, and $y = a \tan b\theta$

Recall The sine and cosine are periodic functions with a period of 360° or 2π radians. The tangent function has a period of 180° or π radians.

◆ **Example 1**
Graph $y = \sin\theta$, $y = 2\sin\theta$, and $y = \sin 2\theta$ on the same set of axes.

◆ **Solution**
The graph of $y = a \sin b\theta$ has an amplitude (height above x-axis) of $|a|$ and period of $\frac{360°}{b}$.

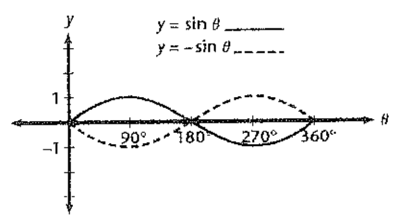


function	amplitude	period
$y = \sin\theta$	1	360°
$y = 2\sin\theta$	2	360°
$y = \sin 2\theta$	1	180°

Check your results with a graphics calculator.

◆ **Example 2**
Graph $y = \sin\theta$ and $y = -\sin\theta$ on the same set of axes.

◆ **Solution**
Notice that the graph of $y = -\sin\theta$ is the reflection of $y = \sin\theta$ across the (horizontal) θ-axis.



Sketch each pair of functions on the same set of axes. Use $0° \le \theta \le 360°$.

1. $y = \cos\theta$, $y = \frac{1}{2}\cos\theta$

2. $y = \cos\theta$, $y = \cos 3\theta$

3. $y = \sin\theta$, $y = -\sin\theta$

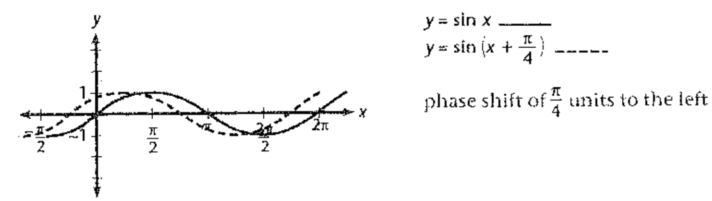
4. $y = \sin\theta$, $y = \sin\frac{1}{2}\theta$

◆Skill EE Graphing functions of the form $y = \sin(x - c) + d$, $y = \cos(x - c) + d$, and $y = \tan(x - c) + d$

Recall The graph of $y = \sin(x - c)$ is a **phase shift** (horizontal translation) of the graph of $y = \sin x$ to the **right** c units.
The graph of $y = \sin x + d$ is a **vertical shift** of the graph of $y = \sin x$ up d units.

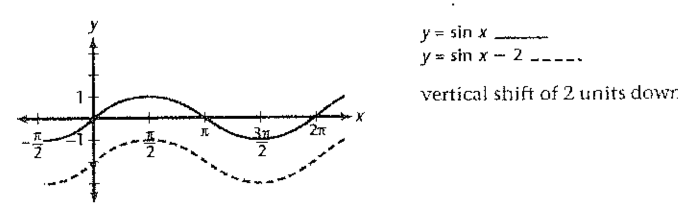
◆ **Example 1**
Graph $y = \sin x$ and $y = \sin\left(x + \frac{\pi}{4}\right)$ on the same set of axes.

◆ **Solution**



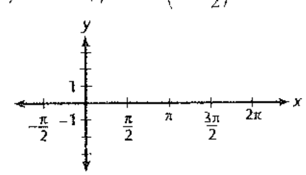
◆ **Example 2**
Graph $y = \sin x$ and $y = \sin x - 2$ on the same set of axes.

◆ **Solution**

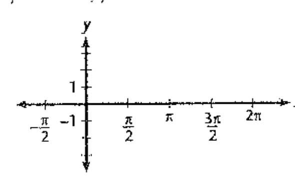


Sketch each pair of functions on the same set of axes. Use $-\frac{\pi}{2} \le x \le 2\pi$.

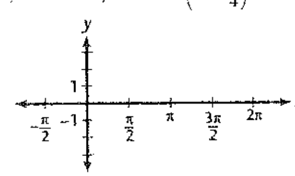
1. $y = \cos x$, $y = \cos\left(x - \frac{\pi}{2}\right)$



2. $y = \cos x$, $y = \cos x + 1$



3. $y = \sin x$, $y = -\sin\left(x + \frac{\pi}{4}\right)$



4. $y = \sin x$, $y = \sin x - 1$