<u>AP Calculus AB Summer Packet</u>

Dear AP Calculus AB Students,

I am so excited to teach you next year!!! AP Calculus is probably my favorite thing to teach!

The goal of this summer packet is to go over a few important skills that you have learned in the past that are frequently used in calculus. In doing so, it will give you a chance to review so that we can cut out several days of review to have more time for Calculus topics and practice exam problems at the end of the year. It is very common for students to do well with the calculus itself, but struggle with the algebra that goes with it. I would like to try to avoid that.

You must show ALL necessary work. The packet will be due the first day of class. Do not do this packet right away, but do not wait until the last minute.

The first portion should be completed WITHOUT a calculator.

The last 2 pages are a circuit (read the directions), and it is intended for you to USE A CALCULATOR. If there are things you don't know how to do, google it – you'll learn some of the things you DO need to know how to do with a calculator. :-)

If you find yourself struggling with some of these problems, please check out the **<u>videos posted in Google Classroom</u>**. Some parts are probably easier than others, but there are resources available if needed. ☺

Feel free to email me over the summer if you have any questions at all. ☺

Mrs. Kincaid
mkincaid@tcswv.org

Calculus - SUMMER PACKET

NAME:___________________________

Summer + Math = $(\text{Best Summer Ever})^2$

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

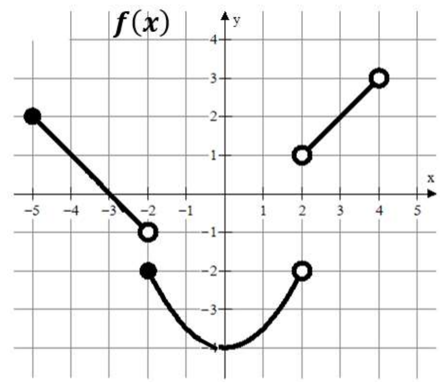
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $f(4) =$

$f(-1) =$ $f(-2) =$

$f(2) =$ $f(3) =$

$f(x) = 2$ when $x = ?$ $f(x) = -3$ when $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

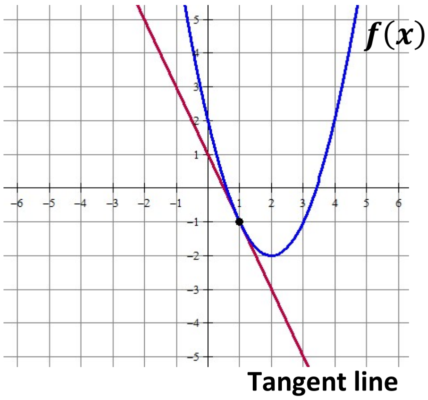
5. slope $= 3$ and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

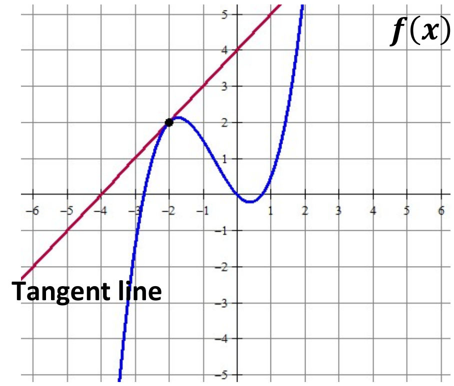
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



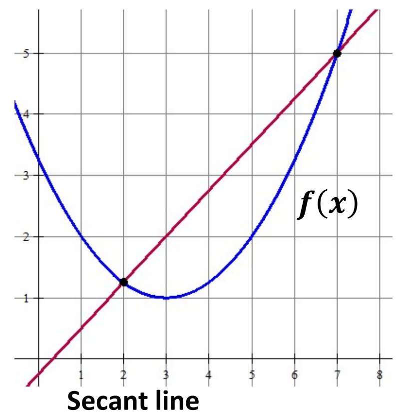
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

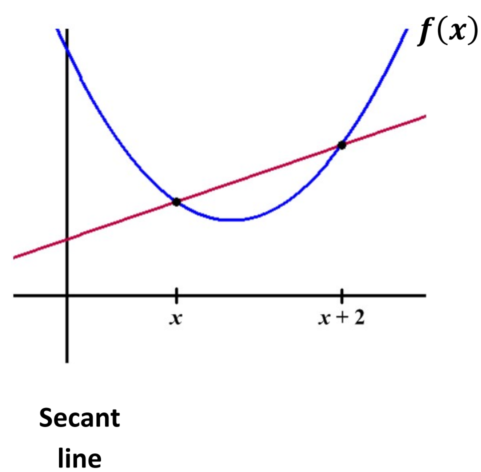
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



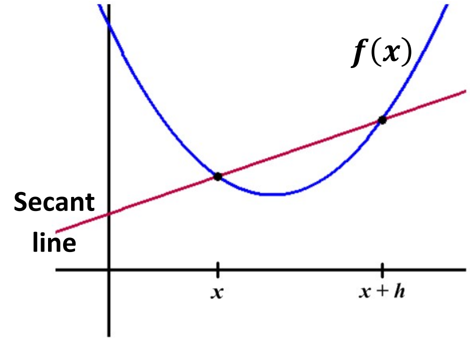
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?

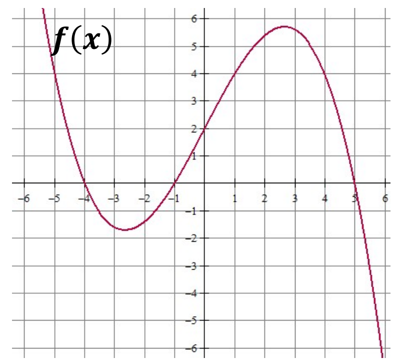


A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

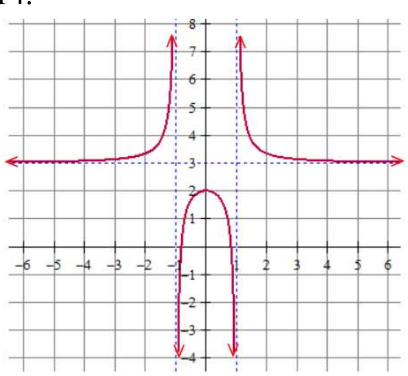
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$



(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



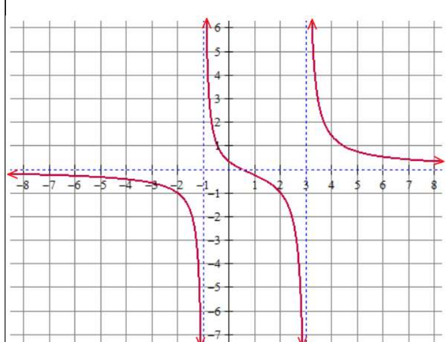
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



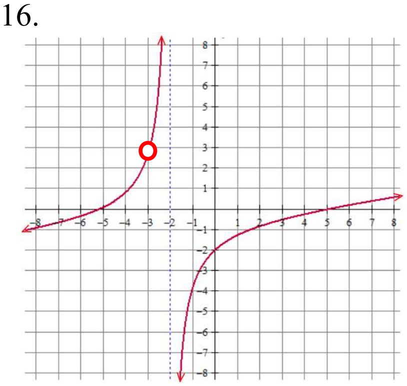
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$	73. $\ln e^7$	74. $\log_3 \frac{1}{3}$
75. $\log_{1/2} 8$	76. $\ln \frac{1}{2}$	77. $27^{\frac{2}{3}}$
78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$	79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$	80. $\left(4a^{5/3}\right)^{3/2}$

If $\boldsymbol{f(x) = \{(3,5),(2,4),(1,7)\}}$ $\boldsymbol{g(x) = \sqrt{x-3}}$ $\boldsymbol{h(x) = \{(3,2),(4,3),(1,6)\}}$ $\boldsymbol{k(x) = x^2 + 5}$**, then determine each of the following.**

81. $(f+h)(1)$	82. $(k-g)(5)$	83. $f(h(3))$
84. $g(k(7))$	85. $h(3)$	86. $g(g(9))$

87. $f^{-1}(4)$	88. $k^{-1}(x)$
89. $k(g(x))$	90. $g(f(2))$

Circuit Training **– Do you know your calculator?** **NAME________________________**

Use your calculator to complete the first problem in the space provided. Circle your answer. Find your answer among the choices. Put #2 in the problem blank. Work that question and proceed in this manner until finished. You may use any of the tools on your calculator to solve these problems.

Answer: 4.272 #1____ Evaluate: $\sqrt[3]{76.5}$	Answer : 1.024 # ______ Solve for x. $\lvert x^3-4x\rvert = 7-x$ To advance in the circuit, find the sum of the two solutions.
Answer: 0.813 # _____ Find the minimum value of the function $h(x)=1+x+e^{x^2+3x}$.	Answer: 4.277 # ______ Let $f(x)=e^{x-4}+2.5x-11.7$. Find the zero of the function.
Answer: 1.527 # _____ Solve for x on the closed interval [2,4]. $\dfrac{20}{3+e^{\tan x}}=5.3$	Answer: –0.144 # _____ Solve for x. $(2x+1)^{-2}=10-e^{x^2+2}$ There are two solutions. To advance in the circuit, find the smallest solution.
Answer: 6.990 #_____ If $f(x)=\ln(x+4)$ and $g(x)=\tan(x^2)$, find $f(g(3.2))$.	Answer: 1.682 # _____ Evaluate: $\ln(5.86)$
Answer: 0.456 # ______ If $h(x)=\begin{cases} x\sec x, & x\le 1 \\ x\tan^{-1}x, & x>1\end{cases}$ find $h(0.9)$ and $h(1.1)$. To advance in the circuit, find the largest of the two values.	Answer: –1.256 # _____ If $f(x)=x^5-2x^4+\sin^2 x+k$, find k so that $f(2.1)=1.212$.

Answer: 1.622 # _____ Solve for x. $\frac{2}{x+2} - \frac{7}{x-5} = 10$ There are two solutions. To advance in the circuit, find the positive solution.	Answer: 1.768 # _____ If $f(x) = 4.5x^3 - 3.2x^2 - \sin x$, find $f(1.5)$.
Answer: –0.321 # _____ Solve for x. $\lvert 3x-4 \rvert = 2.5\sqrt{3-x}$ There are two solutions. To advance in the circuit, find the solution closest to zero.	Answer: –1.478 # _____ If the radius of a cone is 0.9 inches and the height is twice the radius, what is the volume (in inches^3) of the cone? $\left(V = \frac{1}{3}\pi r^2 h\right)$
Answer: 4.245 # _____ Evaluate: $(51.4)^{3/7}$	Answer: 1.448 # _____ If the volume of a sphere is 4.5 m^3, find the radius of the sphere. $\left(V = \frac{4}{3}\pi r^3\right)$
Answer: 2.890 # _____ A remote control plane climbs at takeoff with a slope $m = 0.178$. How far off the ground is the plane when it has traveled 24 feet in the horizontal direction after takeoff?	Answer: –0.176 # _____ Find the maximum value of the function $g(x) = \frac{4.3x}{x^2 + 7}$
Answer: 4.194 # _____ If $g(x) = \sin^2(2x)$, find $g(1.2)$.	Answer: 5.411 # _____ Evaluate: $e^{0.52}$