Calculus - SUMMER PACKET

NAME:__________________________

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$	2. $f(x+2) =$	3. $f(x+h) =$

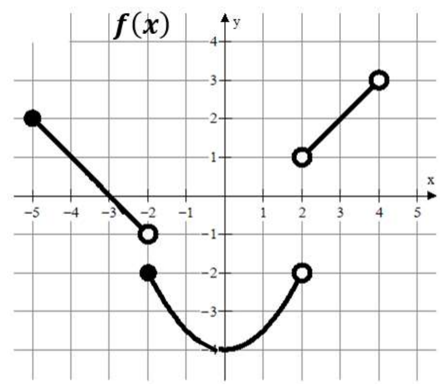
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $f(4) =$

$f(-1) =$ $f(-2) =$

$f(2) =$ $f(3) =$

$f(x) = 2$ when $x = ?$ $f(x) = -3$ when $x = ?$

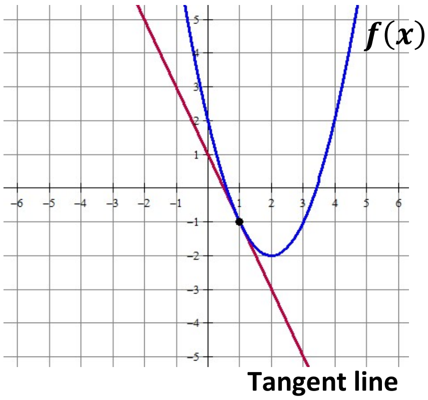


Write the equation of the line meets the following conditions. Use point-slope form.
$\boldsymbol{y - y_1 = m(x - x_1)}$

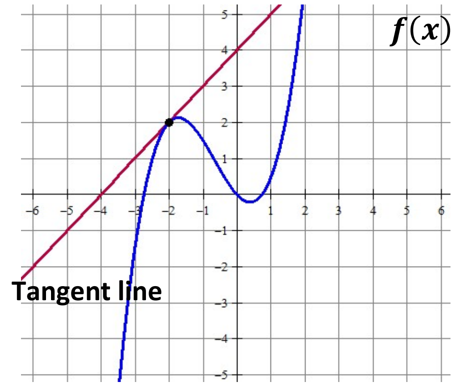
5. slope $= 3$ and $(4, -2)$	6. $m = -\frac{3}{2}$ and $f(-5) = 7$	7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

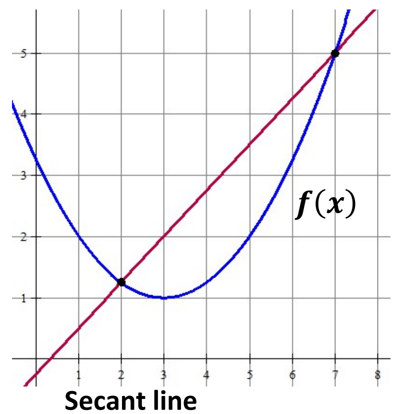


9. The line tangent to $f(x)$ at $x = -2$



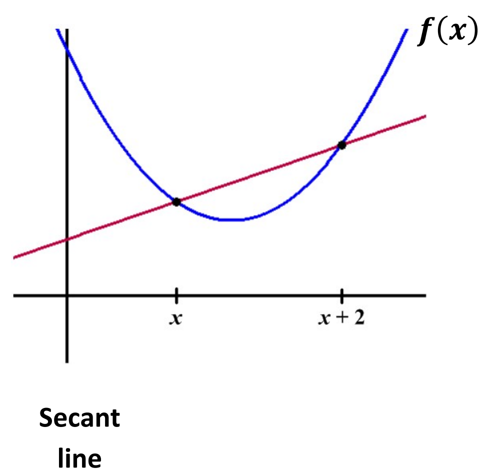
MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

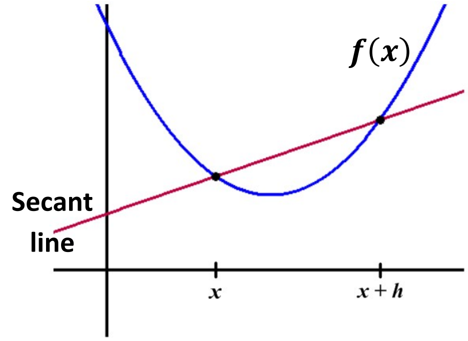
11. Which choice represents the slope of the secant line shown?



A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

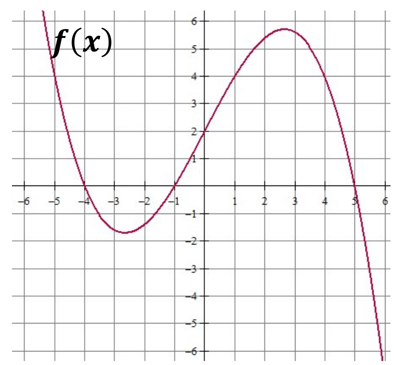
12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

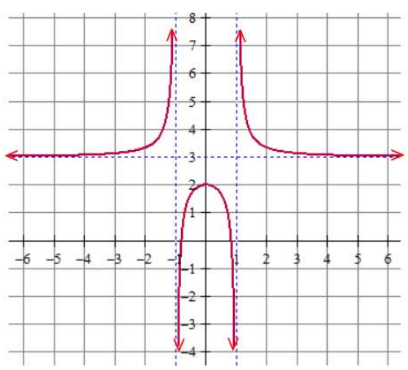


I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



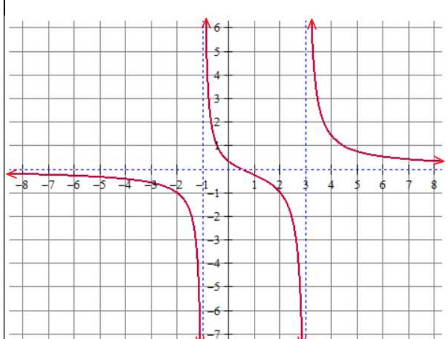
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



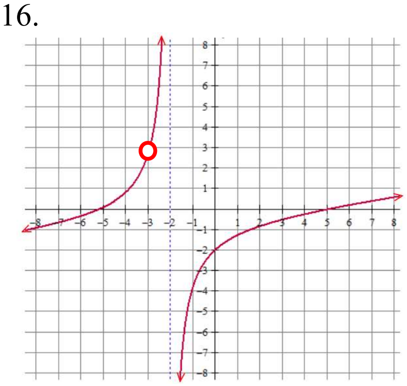
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \leq x \leq 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \leq x \leq 2\pi$, $\tan x = 0$	45. Find x where $0 \leq x \leq 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $\boldsymbol{f(x) = \{(3,5),(2,4),(1,7)\}}$ $\quad$ $\boldsymbol{g(x) = \sqrt{x-3}}$ $\quad$ $\boldsymbol{h(x) = \{(3,2),(4,3),(1,6)\}}$ $\quad$ $\boldsymbol{k(x) = x^2 + 5}$ **, then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f\big(h(3)\big)$

84. $g\big(k(7)\big)$

85. $h(3)$

86. $g\big(g(9)\big)$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k\big(g(x)\big)$

90. $g(f(2))$

CALCULUS

Write your questions here!

Calculator is always in RADIANS!

CALCULATOR TIPS

1. Use the calculator when it is faster
2. Justify your calculator work on Free Response
3. Do NOT round until the very end
4. Round/Truncate to 3 decimal places

$x = 43.58277289$

$x = -0.49927438$

Common Mistakes

Evaluate

$f(x) = x^2 - 4$ at $x = -3$

$f(x) = \frac{x}{2\pi}$ at $x = 7$

$f(x) = x^9 - 4$ at $x = 47$

Trig Functions

Evaluate

$f\left(\frac{\pi}{5}\right) = \csc\theta$

$f\left(\frac{2}{3\pi}\right) = \sin^2\theta$

Window, Trace, Table, ZStand and ZTrig

Evaluate

$f(x) = x^2 - 4$ at $x = 3.2$

$f(\theta) = \tan^{-1}(\theta)$ at $\theta = \pi$

ZFit, Finding Extrema and Roots

Find all Max/Min

$f(x) = x^4 - 3x^3 + x + 3$

Find the zeros

$f(x) = x^4 - 3x^3 + x + 3$

Finding Point of Intersection

Solve

$y = x^3 + 3x - 4$
$y = -x^2 + 4$

Solve Equations

Solve

$x^3 + 3x - 4 = 5$

STORE and RECALL

If $x = \sin\left(\frac{\pi}{7}\right)$, find $3^x - 2\sqrt{x} - 4x$

$f(x) = x^4 + 3x - 4$
$f(x) = -x^2 + 4$

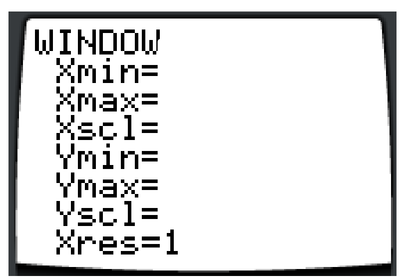
The x coordinate of the left point of intersection is A.
The x coordinate of the right point of intersection is B.

Find $A + B$

Window for Word Problems

Methane is produced in a cave at the rate of $r(t) = e^{\sin\left(\frac{\pi}{4}t\right)}$ liters per hour at time t hours. The initial amount of methane in the cave at time $t = 0$ is 20 liters. At $t = 8$ hours, a pump begins to remove the methane at a constant rate of 1.5 liters per hour.

At what time t during the time interval $0 \leq t \leq 8$ hours is the amount of methane increasing most rapidly?



<u>SUMMARY:</u>

Now, summarize your notes here!

You are allowed to use a graphing calculator for 1-21

Find all extrema and roots for each function.

1. $y = -\frac{9}{10}x^3 - \frac{3}{4}x^2 + 2x + 1$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

2. $f(x) = \frac{e^x - 1}{x^2 - 4}$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

Solve the systems of equations by graphing.

5. $y = -\ln(2x-1) + 3$
   $y = e^{\frac{2}{3}x} - 2$

6. $y = \sqrt{x^2 - 4}$
   $y = \tan^{-1}(x) + 3$

Evaluate the function at the given point.

9. $f(x) = e^{x^2-1}$ at $x = e$

10. $y = \sec(x) + 5x$ at $x = \frac{\pi}{5}$

11. $f(x) = 3x\sqrt{x^2+5}$ at $x = \pi$

12. $y = 2\sin^2(x) + \tan(2x)$ at $x = \frac{\pi}{3}$

Use the STORE feature to evaluate the following.

13. STORE $x = \cot\left(\frac{\pi}{9}\right)$ and use RECALL to find

$$\sqrt{x} + \ln(2x) - e^x$$

14. STORE $x = e^{\pi}$ and use RECALL to find

$$4x - 2\sqrt{x^2+1} + 2^x$$

15. Solve the system of equations below. STORE the x coordinate of the left point of intersection as A. STORE the x coordinate of the right point of intersection as B.

$y = \sin^2(x^2) + 1$
$y = -|2x+1| + 2.5$

Use RECALL to find $A - B$

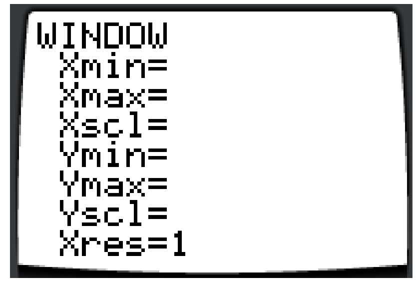
16. STORE the x coordinate of the maximum point as A. STORE the x coordinate of the minimum point as B.

$y = -\frac{2}{5}x^3 - 2x^2 + x + 7$

Use RECALL to find $A - B$

State the WINDOW that allows you to view the function. Answer the question.

17. A tortoise runs along a straight track, starting at position $x = 0$ at time $t = 0$. The tortoise has a velocity of $v(t) = \ln(1 + t^2)$ inches per minute, where t is measured in minutes such that $0 \le t \le 15$.



What is the tortoise's velocity at $t = 2.5$?

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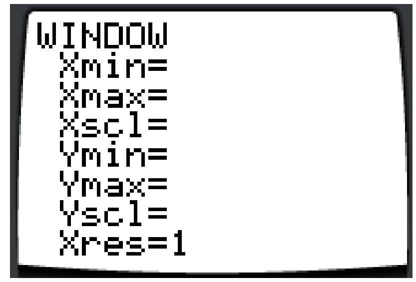
18. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 30-day period. The rate at which the height of the water is rising in the can is given by $s(t) = 2\sin(0.03t) + 1.5$ where $s(t)$ is measured in millimeters per day and t is measured in days.



When will the rate of change of the height be 2 mm/day?

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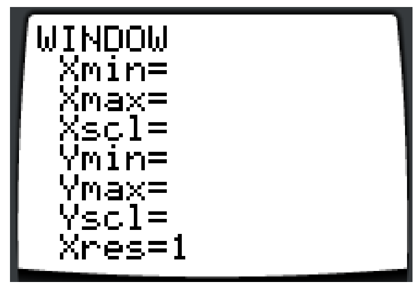
19. For $0 \le t \le 6$, a particle is moving along the *x*-axis. The particle's position, $x(t)$, is not explicitly given. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ in units per second^2.

(NOTE: Acceleration can be positive or negative!)



What is the particle's maximum acceleration?

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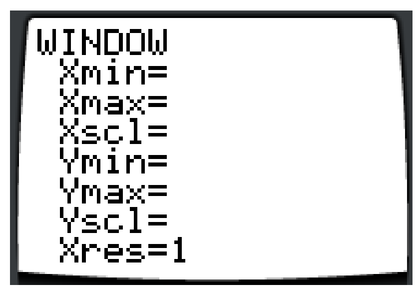
20. The temperature on New Year's Day in Mathlandia was given by by $T(H) = -5 - 10\cos\left(\frac{\pi H}{12}\right)$ where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight $0 \le H \le 24$.



Find $T(12)$ and explain what it means in this context.

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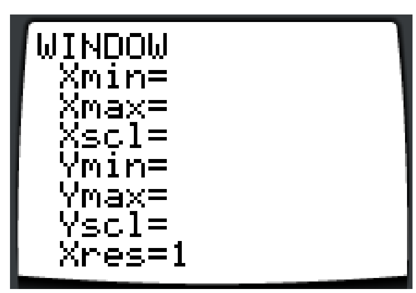
21. A hospital patient is receiving a drug on an IV drip. The rate at which the drug enters the body is given by $E(t) = \frac{4}{1+e^{-t}}$ cubic centimeters per hour. The rate at which the body absorbs the drug is given by $D(t) = 3^{\sqrt{t}-1}$ cubic centimeters per hour. The IV drip starts at time $t = 0$ and continues for 8 hours until time $t = 8$.



Is the amount of drug in the body increasing or decreasing at $t = 6$?

You are allowed to use a graphing calculator for 1-4

MULTIPLE CHOICE

1. Find the value of x for which the graphs of $f(x) = \frac{1}{2}e^{x-4}$ and $g(x) = 3\sqrt[3]{x}$ have $f(x) = g(6)$.

   (A) -1.761
   (B) 0.35
   (C) 2.134
   (D) 5.451
   (E) 6.389

2. Find the minimum value of the function $f(x) = \ln(x) + \sin(x)$ on the interval $\left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$.

   (A) 0.465
   (B) 0.526
   (C) 0.785
   (D) 1.145
   (E) 1.605

3. If $f(x) = -\frac{x^2}{x^3-8}$, how many values of c such satisfy the condition $f(c) = 0$?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

4. Which of the following statements about the function $y = x^3(3-x)$ is true?

   I. The function has an absolute maximum.
   II. The function has an absolute minimum.
   III. The function has a relative minimum.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) I and III

You are allowed to use a graphing calculator

FREE RESPONSE

Your score: ____ out of 5

An online retailer has a warehouse that receives packages that are later shipped out to customers. The warehouse is open 12 hours per day. On one particular day, packages are delivered to the warehouse at a rate of $D(t) = 300\sqrt{t} - 3t^2 + 75$ packages per hour. Packages are shipped out at a rate of $S(t) = 60t + 300\sin\left(\frac{\pi}{6}t\right) + 300$ packages per hour. For both functions, $0 \le t \le 12$, where t is measured in hours. At the beginning of the workday, the warehouse already has 4000 packages waiting to be shipped out.

1. What is the rate of change of the number of packages in the warehouse at time $t = 10$?

2. What is the rate of change of packages shipped out of the warehouse when the rate of change of packages delivered to the warehouse on this day is a maximum?

3. During what time interval(s) is the rate of packages being delivered to the warehouse greater than rate of packages being shipped out of the warehouse?